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Relationship Between Transition and Modes of Instability in Supersonic Boundary Layers

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Abstract

The relationship between the predicted transition location and the first and second modes of instability in two-dimensional supersonic boundary-layer flow on a flat plate is examined. Linear stability theory and the N-factor criterion are used to predict transition location. The effect of heat transfer is also studied; the results demonstrate that the transition reversal phenomenon can be explained by the opposite effect of cooling on the first and second modes of instability. Compressibility is destabilizing at free-stream Mach numbers of 2 to 3.5. The predicted transition location is due to the oblique first modes of instability, up to free-stream Mach numbers between 6 and 6.5. At higher Mach numbers, the predicted transition location is due to a combination of two-dimensional first and second modes of instability.

1. Introduction

A relationship exists between different laminar flow instability mechanisms in low- and high-speed boundary-layer flows on aerodynamic surfaces and the breakdown of the laminar flow to turbulence. However, the details of this relationship are not well understood, particularly for high-speed flows. The most common approach for relating instability to transition is the N-factor method, in which transition is assumed to occur whenever the integral of the linear growth rate of any instability wave reaches a certain value that is close to 9 in two-dimensional (2-D) flows. In high-speed flows, the instability is complicated by the possible existence of higher modes of instability, in addition to the single mode of instability (the Tollmien-Schlichting (T-S) wave) that exists in low-speed flows.

Mack[1] pointed out that in boundary layers the mean flow, relative to the disturbance phase velocity, can be supersonic. Mack determined through numerical studies that whenever the relative flow is supersonic over some portion of the boundary-layer profile, an infinite number of wave numbers corresponds to a single phase velocity. He called the additional inviscid disturbances "the higher modes" and showed that for 2-D disturbances the first of the additional modes is the most unstable. Mack called the first of the higher modes the second mode, which is also referred to in the literature as the Mack mode. The originally known mode that corresponds to the mode in incompressible flow is called the first mode. Through direct numerical calculations, Mack revealed that three-dimensional (3-D) first-mode disturbances are more unstable than 2-D first-mode disturbances at all supersonic Mach numbers.

The first mode of instability is characterized by the fact that its growth rate peaks at relatively low frequencies when compared with the second and higher modes. In general, the higher the mode, the higher the frequency at which its growth rate peaks. Except for the second mode, the higher modes are usually damped at finite Reynolds numbers. Mack[2] demonstrated the capability to distinguish between different modes when the number of oscillations in a disturbance quantity eigenfunction (such as a pressure disturbance eigenfunction) which corresponds to a certain eigenvalue is considered. However, in the regions where the modes merge with each other, the determination of whether the mode is, for example, a first or a second mode is difficult. The merging of unstable modes takes place at Mach numbers that are higher than approximately 4.8 in adiabatic boundary layers at wind-tunnel temperatures; this value increases with cooling. The second mode becomes amplified at a Mach number of approximately 3.5 in adiabatic boundary layers. In the present work, we examine the relationship between transition and modes of linear instability in both adiabatic and cooled (or heated) high-speed flows over a flat plate.

2. Notation and Assumptions

In the calculations and results presented in this work, the spatial stability theory is used. Therefore, the frequency of the disturbance is real, whereas the spatial eigenvalue is complex. The growth rate of the wave is denoted by $-\alpha_i$; its streamwise wave number is denoted by α_r. In addition, $\alpha = \alpha_r + i\alpha_i$, where $i = \sqrt{-1}$ is made nondimensional with respect to the length scale $\delta_r^* = \sqrt{\nu_\infty^* x^* / U_\infty^*}$, such that $\alpha = \alpha^* \delta_r^*$. The dimensional free-stream kinematic viscosity is denoted by ν_∞^* and is equal to $\mu_\infty^* / \rho_\infty^*$, where μ_∞^* is the dimensional free-stream dynamic viscosity and ρ_∞^* is the dimensional free-stream density. The spanwise wave-number parameter B is defined as $B = 1000\beta/R$ and is fixed for the same physical wave as it propagates downstream. The nondimensional spanwise wave number is $\beta = \beta^* \delta_r^*$, and β^* is the dimensional spanwise wave number of the wave. The stability Reynolds number is $R = U_\infty^* \delta_r^* / \nu_\infty^*$. The frequency parameter F is defined as $F = 2\pi f^* \nu_\infty^* / U_\infty^{*2}$, where f^* is the dimensional frequency in cps (Hz) and is related to the dimensional circular frequency ω^* through $\omega^* = 2\pi f^*$. Therefore, $\omega = \omega^* \delta_r^* / U_\infty^*$ is related to F through $\omega = FR$.

In the compressible mean-flow and stability calculations, the Prandtl number Pr and the specific heat at constant pressure C_p are usually constant, and the dynamic viscosity μ and the thermal conductivity κ normally vary with temperature in accordance with the Sutherland formula. Mack[3] presented a formulation for the stability problem that accounts for a variable Pr and a constant C_p. Reed and Balakumar,[4] Bertolotti,[5] and Masad et al.[6] presented formulations and results to account for variable μ, C_p, and κ. Reed and Balakumar[4] and Masad et al.[6] used the conventional linear quasi-parallel stability theory, and Bertolotti[5] used the linear parabolized stability equations (PSE), which account for the leading-order nonparallel effects. The effect of variable fluid properties on the maximum growth rate of instability waves was small. The results presented in the present work were obtained with a constant Prandtl number of 0.72 in both the mean-flow and stability calculations.

In boundary-layer stability analyses, the quasi-parallel assumption is usually invoked, which means that the mean-flow quantities that arise from the growth of the boundary layer are neglected. The quasi-parallel assumption takes the growth of the boundary layer to be small over a wavelength and, consequently, considers the wave motion to be determined by the local boundary-layer profile, so that the nonparallel terms are neglected. By neglecting the nonparallel terms, the quasi-parallel assumption simplifies the partial-differential disturbance equations by allowing use of the normal-mode form of the solution locally to separate the streamwise, spanwise, and temporal variations. Homogeneous, ordinary differential equations result, with the appropriate homogeneous boundary conditions and, therefore, a differential eigenvalue problem. Two categories of techniques can be used to

account for the nonparallel effects: numerical perturbation methods and linear PSE. The effect of nonparallelism on the stability of 2-D compressible boundary layers was studied in the context of numerical perturbation methods by Gapanov[7] and El-Hady.[8] Some nonparallel results with this approach were also presented by Masad et al.[6] and compared with the experimental data of Kosinov et al.[9] With the PSE, Bertolotti and Herbert,[10] Chang et al.,[11] and Chang and Malik[12] accounted for the nonparallel effects on the stability of compressible boundary layers. The results of the two techniques agree well[12], except in the region where the first mode merges with the second mode. The results also show that the nonparallel effects on the growth rates of three-dimensional disturbances can be significant. All of the results presented in this work were obtained with the quasi-parallel assumption.

3. Stability of Cooled Boundary Layers

For air boundary-layer flow over a flat plate, as the free-stream Mach number M_∞ increases, the adiabatic wall temperature also increases. An approximate, but fairly accurate, formula that relates the adiabatic wall temperature to the free-stream temperature and the free-stream Mach number is given by

$$\frac{T_{ad}^*}{T_\infty^*} = 1 + \frac{\gamma - 1}{2}\sqrt{Pr}\, M_\infty^2 \tag{1}$$

where γ is the ratio of the specific heats, the Prandtl number Pr is given by

$$Pr = \frac{\mu_\infty^* C_p^*}{\kappa_\infty^*} \tag{2}$$

and κ_∞^* is the dimensional free-stream thermal conductivity. Relation (1) is based on the assumption that the temperature profile across the boundary layer is a function of only the streamwise velocity. (See Schlichting,[13] pp. 330-334.) In adiabatic boundary layers, this assumption is satisfied when the Prandtl number is unity; therefore, for a nonunity Prandtl number, relation (1) is approximate. The exact variation of the adiabatic wall temperature with the free-stream temperature and the free-stream Mach number can be computed by numerically solving the compressible boundary-layer equations that are subject to the thermal boundary condition $\partial T / \partial y = 0$ at the wall. A comparison between the adiabatic wall temperature calculated from the approximate formula and the exact numerical solution for a range of free-stream Mach numbers is given in table 1.

As shown in table 1, the adiabatic wall temperature reaches very high values at high Mach numbers. Existing metallic and composite materials cannot withstand some of these high temperatures. In such cases, the materials must be thermally protected, which can be

achieved by cooling the surface. Therefore, an evaluation of the effect of cooling on stability and the transition to turbulence in high-speed boundary layers is of practical importance.

The effect of heat transfer on the inviscid and viscous instability of 2-D compressible boundary layers was studied by Mack.[2,14] More recent studies have been conducted by Wazzan and Taghavi,[15] Gasperas,[16] Malik,[17] Vignau,[18] Arnal et al.,[19] and Masad et al.[6] However, these studies focused on the effect of heat transfer on the stability of supersonic boundary layers, rather than on transition.

Cooling thins the boundary layer and modifies the mean-flow streamwise velocity profile by making it fuller. Both of these changes in the boundary layer have a stabilizing effect. Furthermore, although the adiabatic compressible flow has a single, generalized inflection point inside the boundary layer, cooling creates another generalized inflection point near the wall.[6] An increase in Mach number moves the generalized inflection point away from the wall. As the cooling level increases, the two generalized inflection points move closer to each other until they meet and disappear. The minimum cooling level needed to eliminate the generalized inflection points increases as M_∞ increases.

From both a practical and an experimental point of view, the wall temperature is more easily fixed than the heat flux. Therefore, we express the level of heat transfer by specifying the ratio T_w/T_{ad}, where T_w is the actual wall temperature (made nondimensional with respect to T_∞^*) and T_{ad} is the adiabatic wall temperature (also made nondimensional with respect to T_∞^*). If $T_w/T_{ad} = 1$, then the wall is adiabatic; if $T_w/T_{ad} < 1$, then the wall is cooled; if $T_w/T_{ad} > 1$, then the wall is heated. The adiabatic wall temperature T_{ad} is calculated from equation (1). For first-mode waves in supersonic adiabatic boundary layers, Mack[2] found that oblique (3-D) waves are more unstable than 2-D waves. Cooling has a stabilizing influence on the 2-D and 3-D first-mode waves.

The stabilization by cooling of the most amplified (over all frequencies and wave angles) first-mode waves at $R = 1500$ is shown in figure 1 for three wall temperatures. In the adiabatic case, the maximum growth rate decreases monotonically with M_∞. However, when $T_w/T_{ad} = 0.6$, the maximum growth rate increases with M_∞. Figure 1 shows that cooling is more effective in stabilizing first-mode waves at low rather than high M_∞.

As mentioned earlier, the most amplified second-mode waves are 2-D. Although cooling stabilizes first-mode waves, it has the opposite effect on the most amplified (over all frequencies) second-mode waves and destabilizes them. Heating, rather than cooling, stabilizes second-mode waves. Because second-mode waves are significant at high Mach numbers where the adiabatic wall temperatures are already large in terms of existing materials, heating is not practical in stabilizing the flow at high Mach numbers. Figure 1 shows that the effect of cooling on increasing the maximum growth rates of second-mode

waves decreases as M_∞ increases. Note in figure 1 that for the adiabatic case the growth rate peaks at some value of M_∞, which shifts towards a lower M_∞ as the wall is cooled. Shaw and Duck[20] showed that at a free-stream Mach number of 4 and when the cooling level exceeds a certain value, the inviscid maximum growth rate of second-mode waves begins to decrease. The present viscous calculations at the same Mach number with R = 1500 and T_∞ = 120 K show a similar effect (Figure 2). However, viscous calculations at R = 1500 and at a Mach number of 10 showed that this reversal in the effect of cooling does not take place even with cooling levels as low as T_w / T_{ad} = 0.02 at T_∞ = 50 K (Figure 3) and T_w / T_{ad} = 0.01 at T_∞ = 300 K (Figure 4).

The reference length scale here is $\delta_r^* = \sqrt{\nu_\infty^* x^* / U_\infty^*}$; however, if the displacement thickness is used as a reference length scale, then the length scale will decrease by cooling the surface. Consequently, the rate of change of growth rates with the cooling level will decrease in comparison with the use of δ_r^* as a reference length scale. If the displacement thickness is used as a reference length scale, then the results show that the second mode is nearly unaffected by cooling, which is demonstrated clearly by Arnal.[21] When the amplification factors (N factors) are computed, the results do not depend on the reference length scale.

When the variation of the growth rates with frequency F or the neutral curves in the F–R domain for adiabatic flow over a flat plate is considered, the first mode merges with the second mode. As shown by Mack,[2] up to a certain Mach number the merging takes place while both waves are stable, but at Mach numbers higher than that value, the merging occurs while both modes are unstable. Mack,[2] Arnal et al.,[19] and Arnal[21] have shown that cooling increases the Mach number at which the merging takes place while the first- and second-mode waves are stable; that is, cooling tends to divide the high Mach number, single-region neutral curve into two distinct regions.

4. Transition in Cooled Boundary Layers

The decrease in the effectiveness of cooling on changing the stability characteristics at high Mach numbers is in qualitative agreement with some of the experimental data. Experimental results (Potter[22]) show clearly that the change in the transition Reynolds number at high Mach numbers is slight when compared with the change at low Mach numbers. In fact, some experimental results show that heating or cooling the wall at hypersonic Mach numbers has little or no effect on the location of transition (Deem and Murphy,[23] Sanator et al.,[24] and Krogmann[25]). The experimental work of Demetriades[26] (in which the effect of wall cooling on the boundary layer over a cone at a Mach number of 8 was studied) was the first to show that second-mode disturbances are destabilized by cooling and

shift towards higher frequencies. The experimental data of Lysenko and Maslov[27] at $M_\infty = 4$ clearly demonstrate the destabilizing effect of cooling on second-mode waves. The stabilization of first-mode waves and the destabilization of second-mode waves by cooling was also demonstrated by Stetson and Kimmel[28] for flow past a cone at a free-stream Mach number of 8.

A phenomenon that was observed and reported even in early experimental studies on the effect of cooling in supersonic boundary layers is transition reversal and re-reversal. At moderate Mach numbers between supersonic and hypersonic values and at hypersonic Mach numbers, cooling the wall resulted in a downstream shift in the location of transition in wind-tunnel experiments. However, further cooling shifted this location upstream, which is called transition reversal. Cooling the wall even more caused the transition location to start to move downstream again, which is called transition re-reversal. (See Potter[22].) This phenomenon was noticed by many experimentalists, including Wisniewski and Jack,[29] Sheetz,[30] and Richard and Stollery.[31,32] Van Driest and Boison[33] demonstrated that the effects of roughness, coupled with cooling, may lead to transition reversal. Lysenko and Maslov[34] experimentally explained one of the causes of both transition reversal and transition re-reversal at Mach numbers of 3 and 4. They showed that the decrease in the transition Reynolds number (transition reversal) by cooling is caused by roughness that is created by frost. The increase in the transition Reynolds number (transition re-reversal) by additional cooling was attributed by Lysenko and Maslov to the change of the frost structure, which decreases the roughness by forming a smooth film of frost on the cooled surface. In all tests carried out by Lysenko and Maslov[34] in wind tunnels, transition reversal and transition re-reversal occurred at a wall temperature in the range of 80 to 190 K. The formation of frost in the hypersonic wind-tunnel experiment of Richards and Stollery[32] was observed with transition reversal and re-reversal. However, Richards and Stollery[32] believe that the formation of frost has only a small effect on the movement of the transition location.

The stabilization of the first-mode waves and the destabilization of the second-mode waves by cooling also offer another explanation for transition reversal over a range of Mach numbers, but still do not explain transition re-reversal. Wazzan and Taghavi[15] indicated that the inviscid characteristics of the higher modes may provide an answer to the phenomenon of transition reversal in cooled, compressible boundary layers. When the first mode is responsible for transition, cooling the surface delays transition. However, cooling the surface destabilizes the second mode. Beyond a certain level of cooling, the second mode causes transition, and further cooling causes the location of transition to move upstream (figure 5(a)). This scenario may occur at flight conditions at which the temperatures are, in

general, not as low as those in a wind tunnel. Transition reversal was observed by Merlet and Rumsey[35] on cones in supersonic free-flight tests.

The experimental data give contradictory results on the relationship between wall cooling and the transition Reynolds number at high Mach numbers. Some experimental studies on flat plates and sharp cones report no significant influence of cooling on the transition Reynolds number. However, other experimental results show that cooling has marked influence on the transition Reynolds number at hypersonic Mach numbers. A short, intensive review of the experimental studies on the effect of cooling on boundary-layer transition is given by Potter.[22] The results in figure 5(a) indicate clearly that by applying cooling to the flat plate, the instability mode responsible for transition switches from a first to a second mode. As the flat plate is cooled, the predicted transition location moves downstream and, with sufficient cooling, starts to move upstream (transition reversal). In figure 5(a) at a Mach number of 5, the switch from the first to the second mode of instability as the cause of transition and, consequently, transition reversal occurs at a level of surface temperature between $(T_w / T_{ad}) = 0.7$ and $(T_w / T_{ad}) = 0.8$. This level is expected to decrease as the free-stream Mach number decreases. In fact, recent calculations by Mack[36] at a free-stream Mach number of 3 and a free-stream temperature of 217 K show that the reversal shown in figure 3 takes place at $T_w / T_{ad} \approx 0.6$. Figure 5(a) clearly shows that when both modes of instability are considered the effect of cooling on changing the predicted transition location becomes less significant. The frequencies and spanwise wave numbers that correspond to the data of figure 5(a) are shown in figures 5(b) and 5(c), respectively. We note here that although the maximum growth rate of second-mode waves decreases when the cooling level exceeds a certain high value (Figure 2), a high level of cooling does not cause a transition re-reversal when transition is assumed to occur as the result of a single frequency. The maximum growth rates in figure 2 are associated with very high frequencies that do not contribute to transition, except within an envelope method in which the local maximum growth rate (over all frequencies) is integrated with respect to R to compute the N factor. Only in that context we can get a transition re-reversal at high levels of cooling.

Lysenko[37] presented results similar to those presented in figure 3, but at a Mach number of 4 for flow past a cone. Lysenko's results were obtained by solving the stability equations with the Dunn-Lin[38] approximation.

In figure 5, the points where the actual calculations were made are denoted by circles; the circles are joined by straight lines. The calculations were performed by first fixing the frequency F and the spanwise wave number parameter B and then computing the location (the Reynolds number) where the N factor reached a value of 9. With B fixed, F was then varied, and the new location where the N factor reached 9 was computed, etc. The value of F

that resulted in the lowest Reynolds number at which the N factor reached 9 was then fixed, and B was varied to compute the lowest Reynolds number at which N reached 9. With the computed value of B fixed, F was varied, etc., until both F and B remained fixed within preset tolerances. The tolerance (step) in F was 10^{-6}, and the step in B was 10^{-3}. Because the most amplified second-mode waves are 2-D, a maximization over B is not necessary when these waves are considered. A similar approach is used for the results in figure 6, which is discussed in the next section.

5. Transition in Adiabatic Boundary Layers

A significant result seen in figure 5(a) is that for adiabatic flow with a free-stream Mach number of 5 the predicted transition is caused by the first mode of instability, even though the maximum growth rate of second-mode waves at this Mach number exceeds that of first-mode waves by a large amount (figure 1). The reason for this difference is that the streamwise extent of the unstable range of first-mode instability waves is much longer than that of second-mode waves, although the maximum growth rates of second-mode waves are larger than those of first-mode waves at hypersonic Mach numbers. Therefore, the integrated growth rate (N factor) of first-mode waves reaches the value 9 before that of second-mode waves.

The value of the free-stream Mach number (in adiabatic flow) at which the cause of transition becomes the second mode of instability is of interest. To find this value, the predicted transition location (caused by both first and/or second modes of instability) was computed with the N-factor criterion over a range of Mach numbers that extends from 0 to 7. The results are shown in figure 6(a). The squares in figure 6 indicate conditions where transition is caused by a combination of the 2-D first and second modes of instability. These two modes are assumed to be continuous, which can be justified by considering the variation of the number of oscillations in the eigenfunctions of these modes with the frequency. At the selected free-stream temperature of 150 K and a Prandtl number of 0.72, the predicted transition is caused by a combination of 2-D first and second modes of instability at a free-stream Mach number between 6 and 6.5. The experimental results of Stetson and Kimmel[28] show that second-mode waves are responsible for transition at hypersonic Mach numbers. In figure 6(a), the N-factor criterion predicts that compressibility destabilizes at free-stream Mach numbers in the range of 2 to 3.5. In this range, an increasing Mach number shifts the predicted transition location upstream. This result is significant for laminar flow control (LFC) applications. For supersonic transport, the optimal Mach number appears to be approximately 2 for LFC consideration. Furthermore, this result is in agreement with the experimental data of Sanator et al.,[24] which show a decrease in the transition Reynolds

number between the Mach numbers of 2 and 3.5, followed by an increase in the transition Reynolds number at Mach numbers higher than approximately 3.5. However, a quantitative comparison of the computed and measured transition Reynolds numbers is not valid because the experiments were performed in noisy wind tunnels.

The frequencies and spanwise wave numbers that correspond to the results in figure 6(a) are shown in figure 6(b) and 6(c), respectively. In figures 5 and 6, when the transition location predicted with the N factor criterion moves upstream, the corresponding most dangerous frequency increases. This result occurs when transition is caused by either the first or the second mode of instability. An exception is when transition is caused by a combination of 2-D first and second modes of instability (the squares in figure 6). This relationship is always true in subsonic flows over smooth flat plates (Masad and Malik[39]), as well as in flat plates with a roughness element (Masad and Iyer[40]) and in various thermal and velocity boundary conditions. Note also in figures 5 and 6 that, except for subsonic flows where the most amplified first-mode waves are 2-D, the upstream movement of the transition location is associated with an increase in the value of the spanwise wave-number parameter when the first mode is responsible for transition.

Recent calculations by Mack[36] with e^9 showed that at a free-stream Mach number of 3 and a free-stream temperature of 217 K, the transition Reynolds number for adiabatic conditions was close to 20 million, and the responsible frequency was $F = 6 \times 10^{-6}$. The difference between our results and those of Mack is most likely due to the difference in the free-stream temperature. The calculations of Lysenko[37] for flow past a cone with the stability equations and the Dunn-Lin[38] approximation show results similar to those in figure 6. Results similar to those in figure 6 and for first-mode waves have been reported by Mack[14] and Arnal et al.[19]

6. Conclusions

The relationship between the predicted transition location with the use of the N-factor criterion and the first and second modes of instability in supersonic flow over a flat plate has been studied. The observed transition-reversal phenomenon in cooled supersonic boundary layers can be explained by the stabilizing effect of cooling on the first mode of instability and the destabilizing effect of cooling on the second mode of instability. However, the transition re-reversal phenomenon cannot be explained by these effects, except within an envelope method in which the local maximum growth rate (over all frequencies) of second-mode waves is integrated to compute the N factor. The transition reversal explained in this work is expected to occur under flight conditions in which the temperatures are high enough to prevent the formation of frost on the surfaces. The effect of compressibility is destabilizing at

free-stream Mach numbers in the range of 2 to 3.5. The cause of transition is hypothesized to be the oblique first-mode waves, up to free-stream Mach numbers of 6 or 6.5. At higher free-stream Mach numbers, the cause of transition is hypothesized to be a combination of two-dimensional first and second modes of instability.

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Table 1. Comparison of Directly Computed Adiabatic Wall Temperature With That Calculated From Equation (1) at T_∞ = 120 K and Pr = 0.72

M_∞	Exact	Eq. (1)	Percent error
1	1.16952	1.16971	.016
2	1.67751	1.67882	.078
3	2.52175	2.52735	.222
4	3.69920	3.71529	.435
6	7.04500	7.10940	.914
8	11.70507	11.86116	1.334
10	17.67643	17.97056	1.664

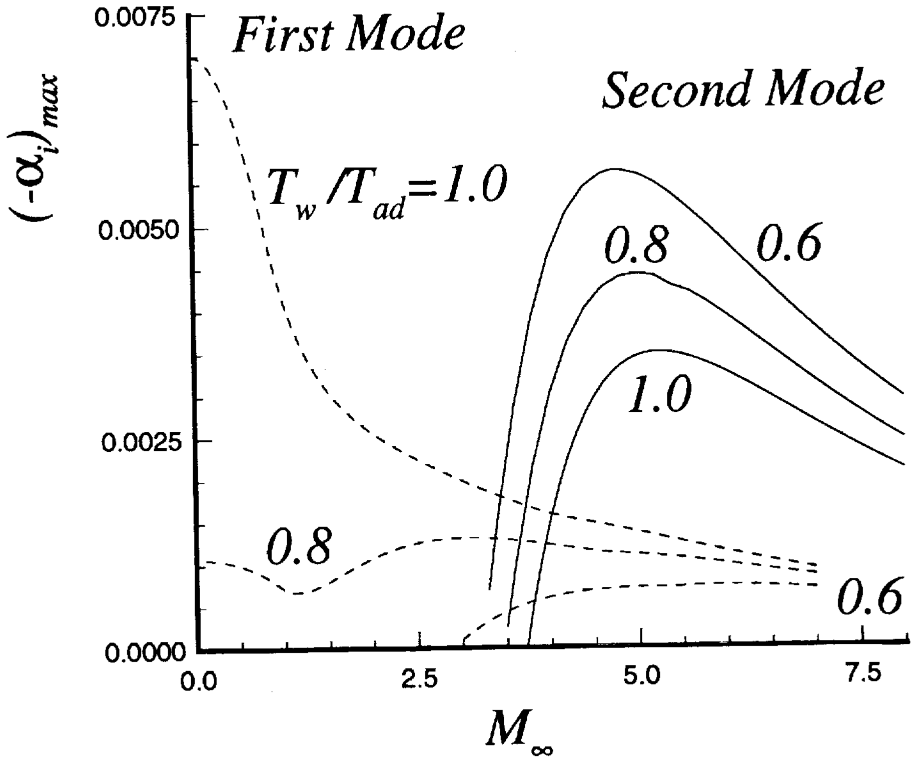


Figure 1. Variation of maximum growth rates of instability waves with M_∞ for three wall temperatures at R = 1500 and Pr = 0.72. (...) First-mode waves with T_∞ = 150 K, and (–) second-mode waves with T_∞ = 120 K.

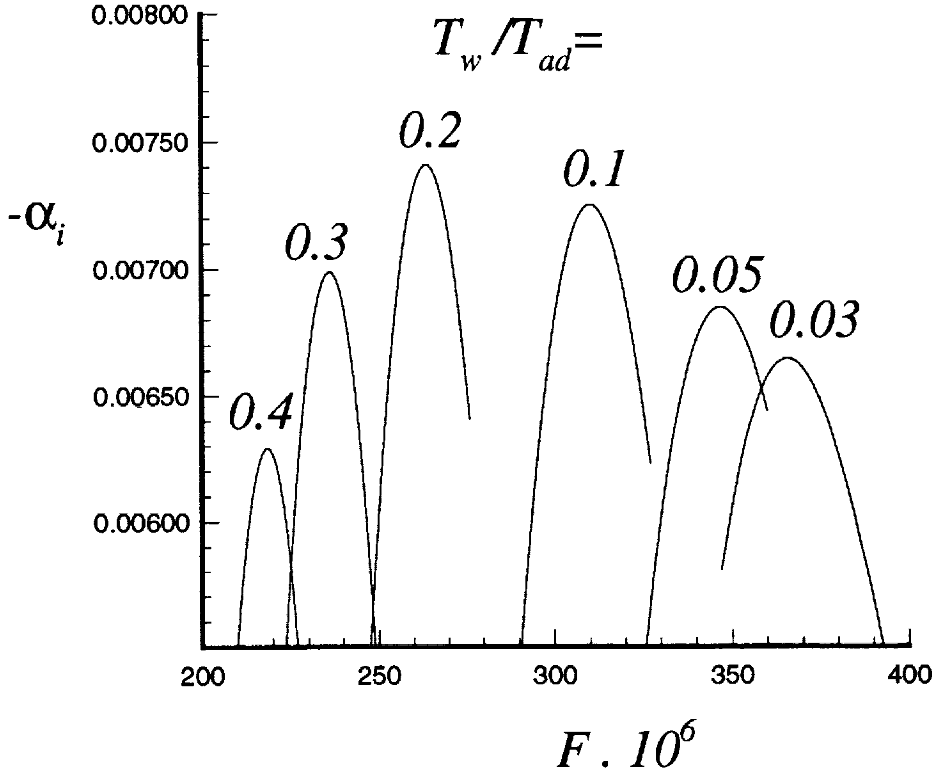


Figure 2. Variation of growth rates of two-dimensional second-mode waves with frequency at $M_\infty = 4$, $R = 1500$, and $T_\infty = 120$ K.

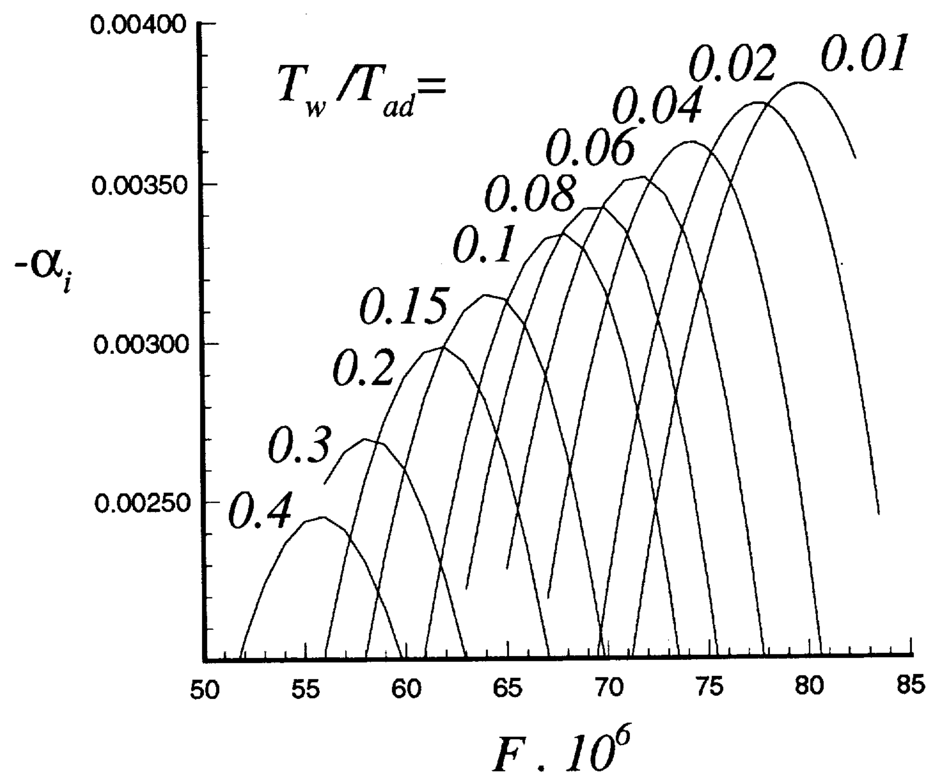


Figure 3. Variation of growth rates of two-dimensional second-mode waves with frequency at $M_\infty = 10$, $R = 1500$, and $T_\infty = 50$ K.

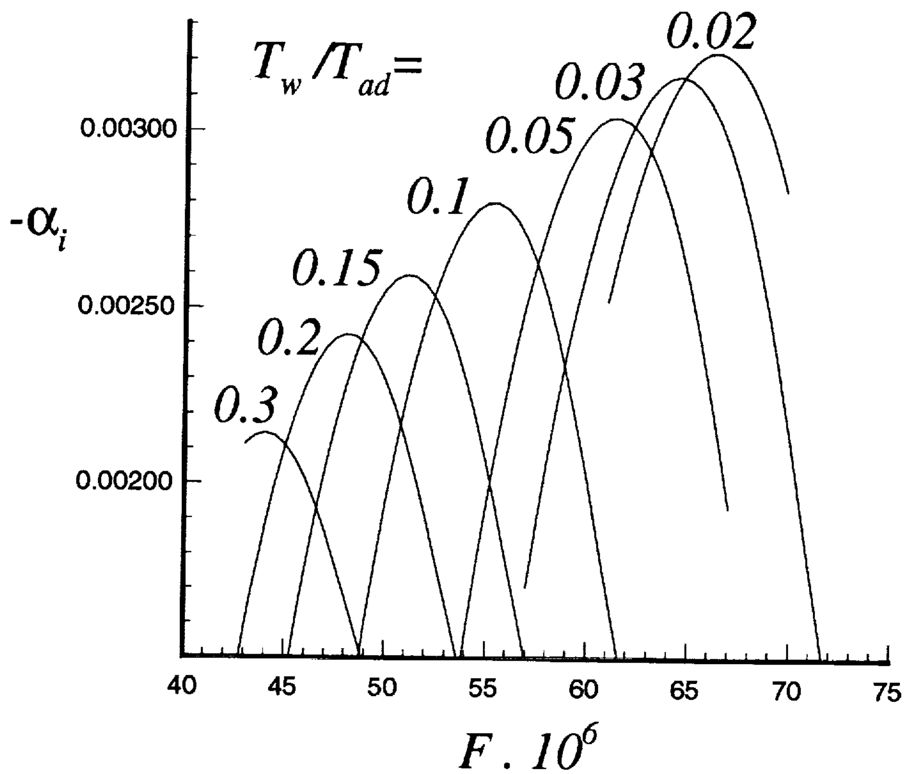
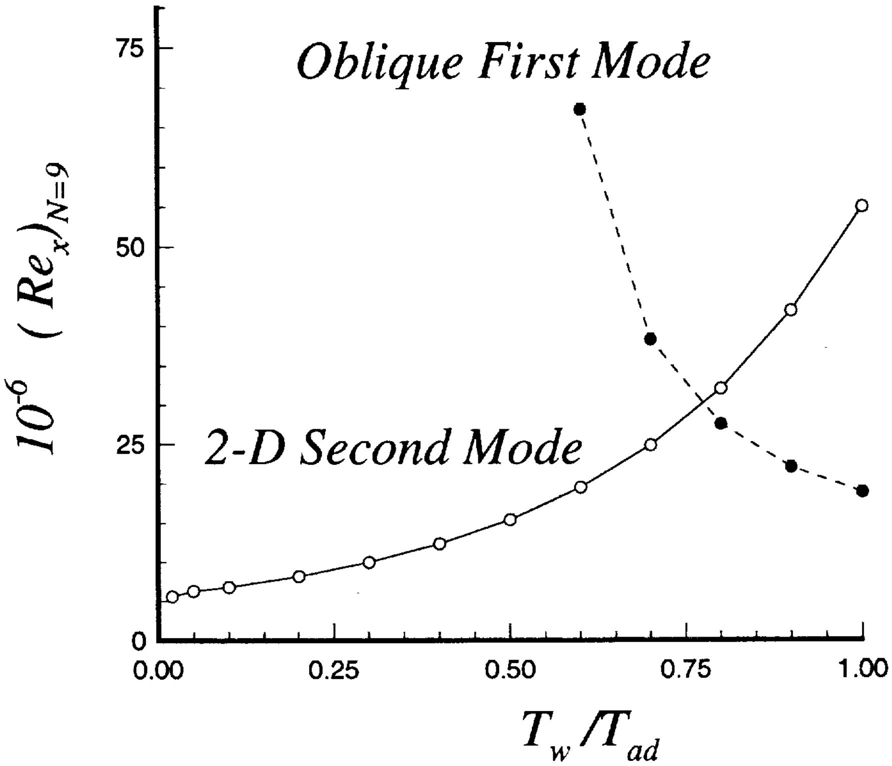


Figure 4. Variation of growth rates of two-dimensional second-mode waves with frequency at M_∞ =10, R = 1500, and T_∞ = 300 K.



(a)

Figure 5. (a) Variation of predicted transition location with wall temperature at $M_\infty = 5, T_\infty = 300$ K, and $Pr = 0.72$. (b) Corresponding frequencies and (c) corresponding spanwise wave numbers.

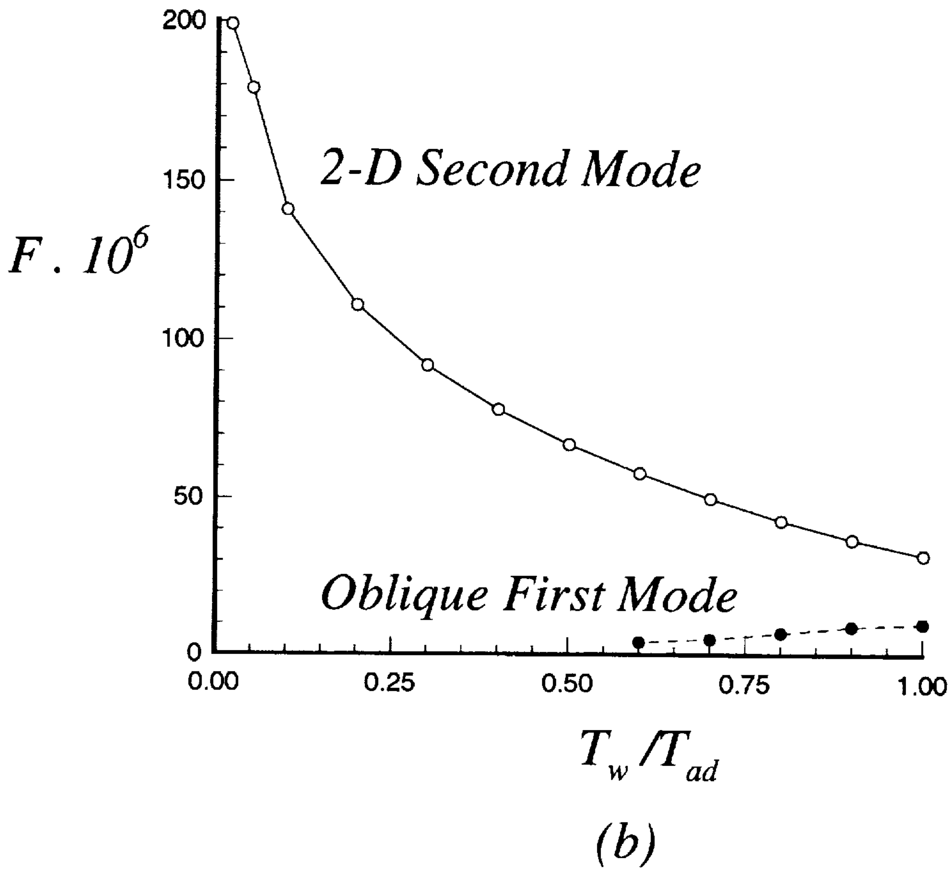
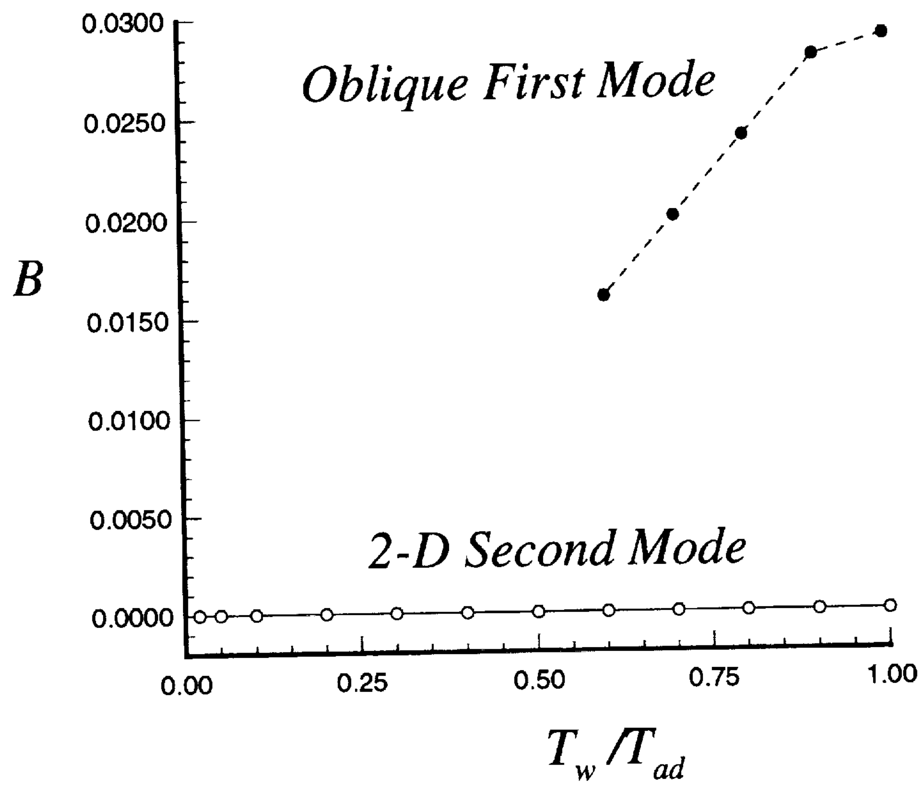
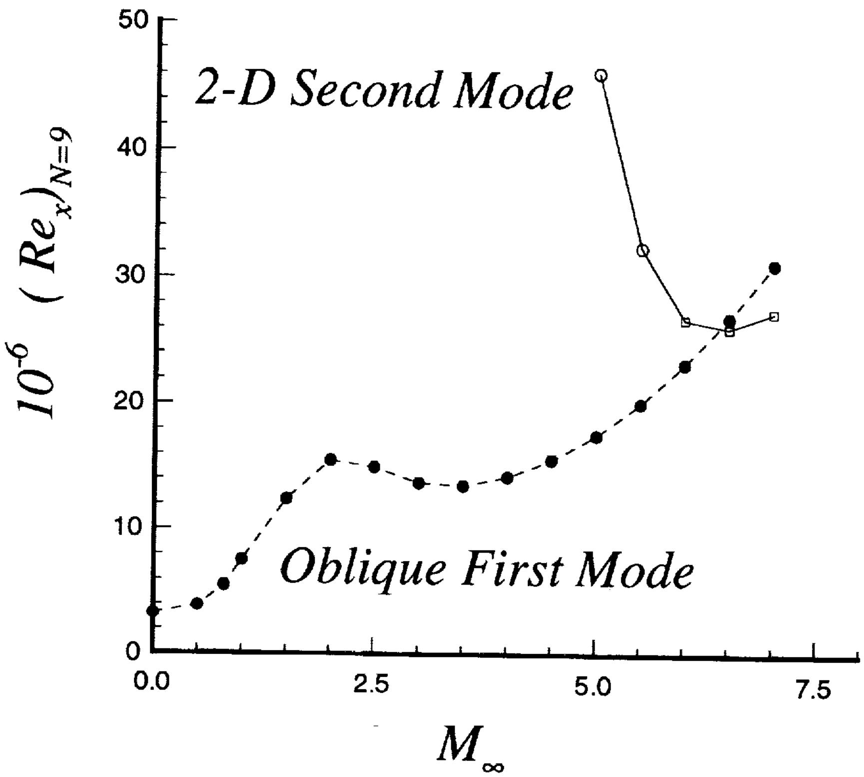


Figure 5. Continued.



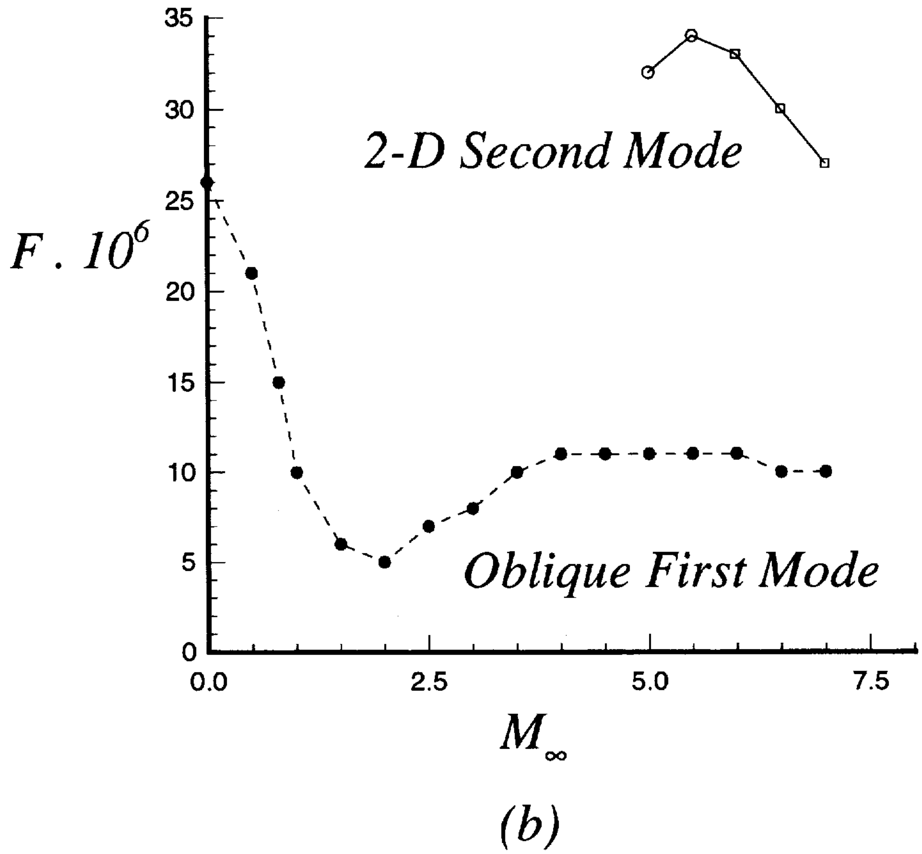
(c)

Figure 5. Concluded



(a)

Figure 6. (a) Variation of predicted transition location in adiabatic flow with free-stream Mach number $T_\infty = 150$ K and $Pr = 0.72$. (b) Corresponding frequencies. (c) Corresponding spanwise wave numbers.



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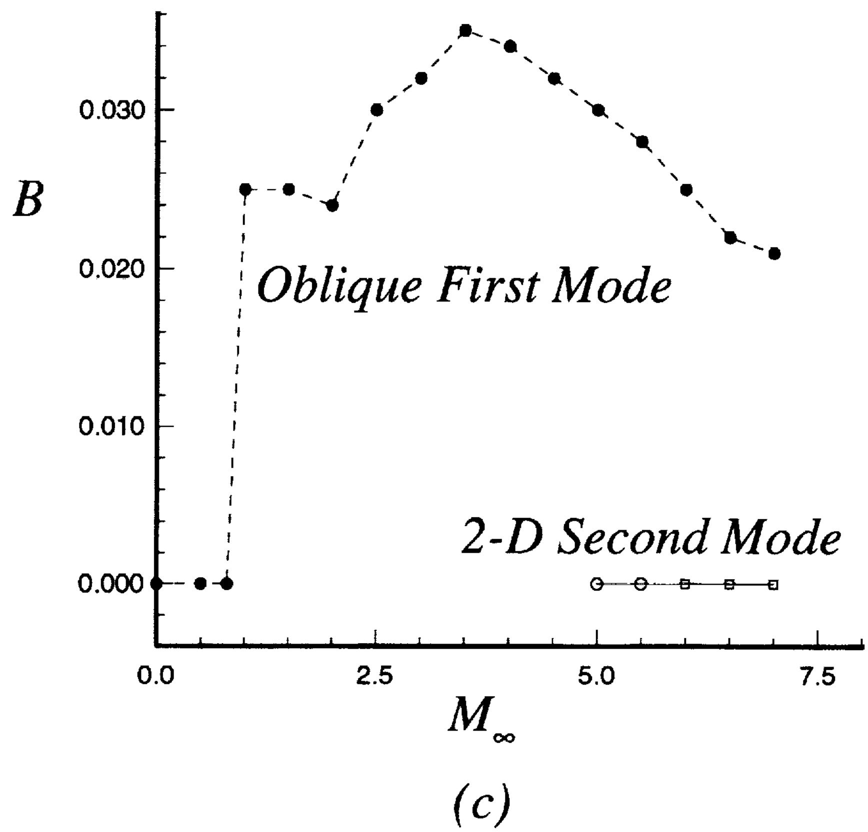

Figure 6. Concluded.

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13. ABSTRACT *(Maximum 200 words)*

The relationship between the predicted transition location and the first and second modes of instability in two-dimensional supersonic boundary-layer flow on a flat plate is examined. Linear stability theory and the N-factor criterion are used to predict transition location. The effect of heat transfer is also studied; the results demonstrate that the transition reversal phenomenon can be explained by the opposite effect of cooling on the first and second modes of instability. Compressibility is destabilizing at free-stream Mach numbers of 2 to 3.5. The predicted transition location is due to the oblique first modes of instability, up to free-stream Mach numbers between 6 and 6.5. At higher Mach numbers, the predicted transition location is due to a combination of two-dimensional first and second modes of instability.

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